

## MODELING OF CIRCULAR SPIRAL INDUCTORS FOR MMICs

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## SUMMARY

A computer model for planar circular spiral inductors considering the distributed nature, the substrate backside metallization and the metallization thickness is presented. The inductance of the structure is calculated using a rigorous evaluation of the two-dimensional inductance integral. The influence of the substrate backside metallization is considered by introducing a mirror coil and all capacitive effects are calculated from high accuracy, frequency dependent formulas for coupled microstrip lines. The theoretical results are discussed and compared to measurements with MIC inductors on dielectric substrate and MMIC inductors on GaAs.

## INTRODUCTION

Simple approximating formulas for calculating planar circular spiral inductors have been described in the literature, /1/-/4/. These formulas do not consider the influence of the distributed nature of the inductors and take into account the influence of the substrate backside metallization only by simple approximations /4/. Many attempts have been described in the literature for modeling the inductance of rectangular spiral inductors (e.g. /5/-/9/), with and without a consideration of the capacitive effects and the ground metallization. Modeling techniques using distributed line elements /8/, /9/ as well as concentrated components /10/ have been described. Up to now only one attempt /11/ was made to model the distributed nature of the circular spiral inductor despite the fact that this inductor has a higher Q-factor than the rectangular spiral inductor and therefore is oftenly used in MMICs. In /11/ a numerical field theoretical analysis has been used to compute the inductance and capacitance matrix of the spiral inductors. As far as it can be seen from the published material, this method leads to a high numerical expense, what means that the computer programs cannot be used directly in CAD programs for the circuit design in desk top computers or workstations.

In this paper a CAD-model for the circular spiral inductor will be presented which calculates the inductance of a planar circular one-turn coil from a rigorous inductance integral, which takes

into account the substrate backside metallization and the capacitive effects and which describes the influence of the metallization thickness by an approximate method. A complete equivalent circuit of the inductor including the losses can be computed from the model for all frequencies up to the first resonant frequency.

## THE FUNDAMENTAL BASIS OF THE COMPUTER MODEL

The circular spiral inductor with  $n$  turns as it is shown in Fig.1a is divided into circular line segments of medium diameters which at their ends are connected to form the circular spiral inductor (Fig.1b). Hentschel /4/ in his doctoral thesis has described an exact evaluation of a two dimensional inductance integral for a circular line segment of width  $w$  and with zero metallization thickness. This formula already takes into account the mutual inductance of the circular line segment with its own current and it is well

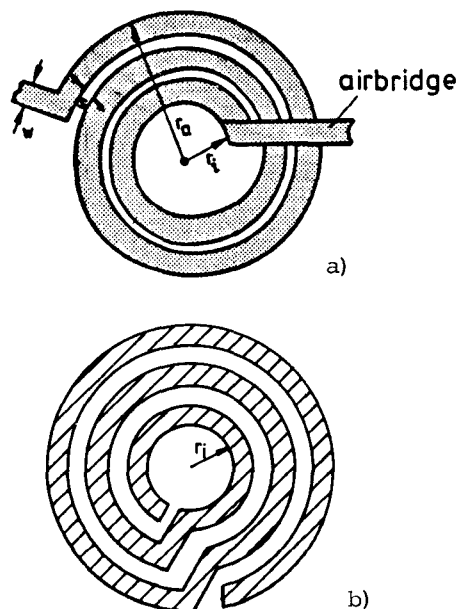


Fig.1: The circular spiral inductor.

suited as a basis for modeling the spiral inductors. Hentschel has also derived an inductance integral for the planar  $n$ -turn circular inductor, but this is a four-dimensional integral which only can be solved on a big computer and therefore cannot be used for CAD applications.

The formula for the circular line segment is taken for calculating the inductance of the  $n$ -turn circular spiral inductor as described above. Additionally the mutual inductances between the different turns have to be computed and have to be added to the total inductance of the coil. This is done by using the well known integral solution for the mutual inductance between two circular line currents (Neumann's formula), which can be solved exactly. Additionally the mutual inductance of a mirror coil is taken into account in the case that the substrate material is metallized on its backside. The influence of the metallization thickness on the inductance is considered approximately by defining an effective line width.

The capacitances between the different turns and between the coil and the ground metallization are calculated from an accurate model [12] of the coupled microstrip line neglecting the curvature effects of the line segments. The resistance of the inductor is calculated from an approximating formula for the full skin effect resistance of a metal strip of rectangular cross section.

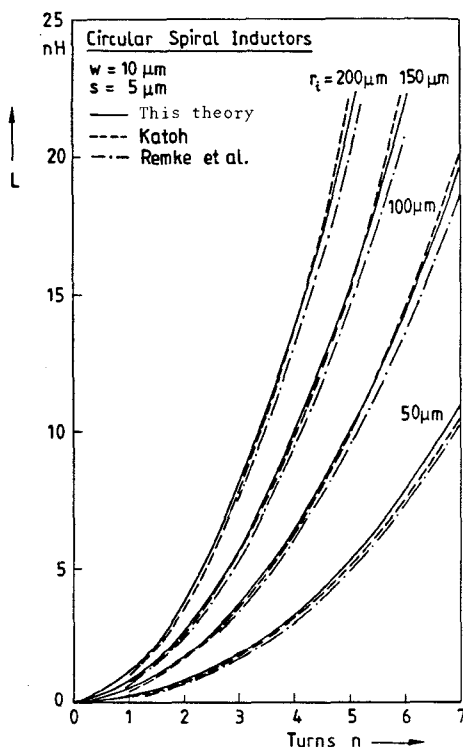


Fig.2: Comparison of different approximations for the static inductance in dependence on the turn number  $n$ .

## RESULTS

Fig 2 shows the calculated static inductance of a spiral inductor without a metallization on the backside of the substrate material, compared with solutions from other formulas [1], [2], [3] which can be used in this simple case. It can be seen from Fig.2 that especially the formula given by Katoh [2] delivers a quite accurate static inductance as long as the radius of the spiral inductor is large and the turn number is larger than one. The agreement between the exact solution of the inductance integral as described above and the formula of Remke [3] is found not to be very good.

In Fig.3 the inductance of a circular line segment on a substrate with and without ground metallization is shown in dependence on the line length (angle  $\Omega_s$ ) and for different values of the GaAs substrate height. Additionally the inductance of a straight line of the same line length is shown in Fig.3. As can be seen from Fig.3, the inductance of the straight line is not linearly dependent on the line length, because of the mutual inductance between the different line segments. Therefore the increase of the inductance is larger than the linear increase.

It also can clearly be seen from Fig.3 that the inductance of the circular line segment is much smaller than that of the straight line segment because with increasing line length the negative mutual inductance between the different line segments of the circular line reduces the total inductance. For opening angles near  $2\pi$  the inductance then increases again, because now the currents in the line segments have the same direction again.

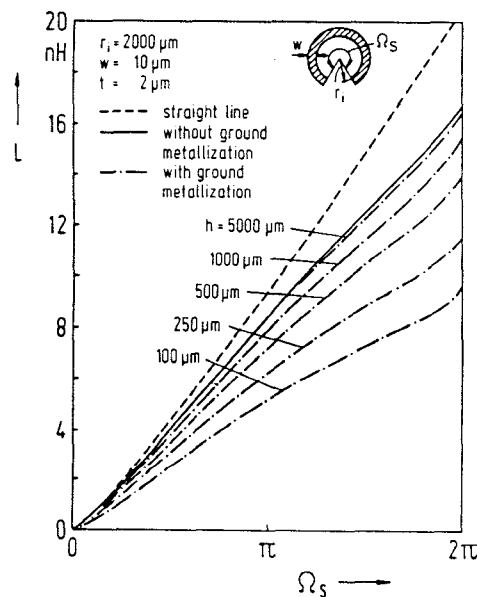


Fig.3: The inductance of a circular line segment and the influence of the ground metallization in dependence on the line length.

If a ground metallization is considered, the inductance decreases with decreasing substrate height (Fig.3). The influence of the ground metallization becomes important for substrate heights smaller than 1 mm (inner radius: 2000  $\mu\text{m}$ ). For a substrate height of 100  $\mu\text{m}$ , the inductance can be reduced to half the value without considering the ground metallization.

As Fig.4 shows, the influence of the ground metallization also is dependent on the inner radius of the spiral inductor. In Fig.4 the inductance of a circular line segment again is shown in dependence on the opening angle  $\Omega$  and for a substrate height  $h=100\text{ }\mu\text{m}$ . For large radii (e.g.  $r_i=1000\text{ }\mu\text{m}$ ) the magnetic flux is distributed over a large area and the ground metallization heavily influences the inductance; if the radius is small (e.g. 100  $\mu\text{m}$ ), the magnetic field of the inductor is concentrated near the line structure and the ground metallization nearly has no influence on the value of the inductance.

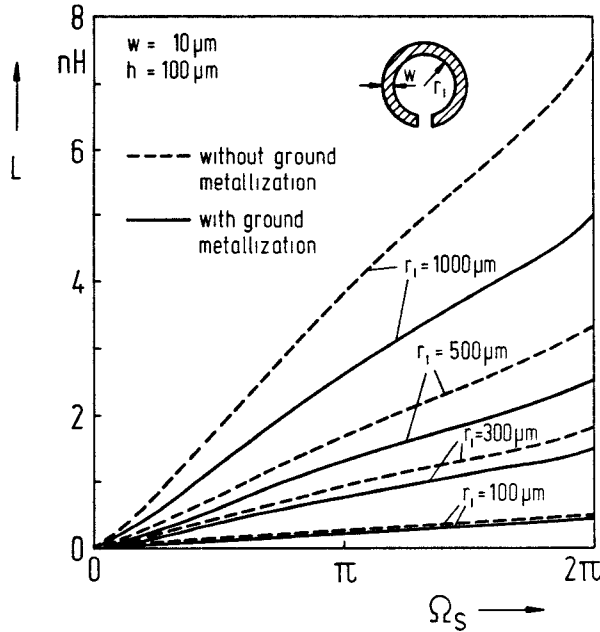


Fig.4: The static inductance of a circular line segment in dependence on the line length and with the inner radius as a parameter.

If an equivalent circuit of the spiral inductor is assumed, which describes the influence of the distributed nature of the structure by a frequency dependent inductance, inductances in dependence on the frequency, as they are shown in Fig.5, can be computed. A one turn inductor nearly has no frequency dependence, the inductance slightly decreases with increasing strip width. If the turn number is increased, the resonant behaviour of the inductance can be recognized; the frequency dependence of the inductance increases with increasing strip width because of the higher capacitive effects between the inductor lines and the ground metallization of the substrate (GaAs). The influence of the slot width on the inductance is not very large.

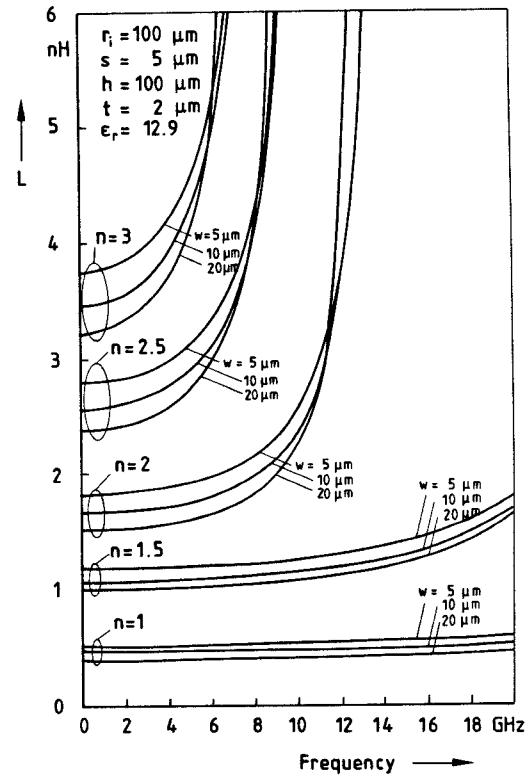


Fig.5: The inductance of a circular spiral inductor in dependence on the frequency and with the line width as a parameter.

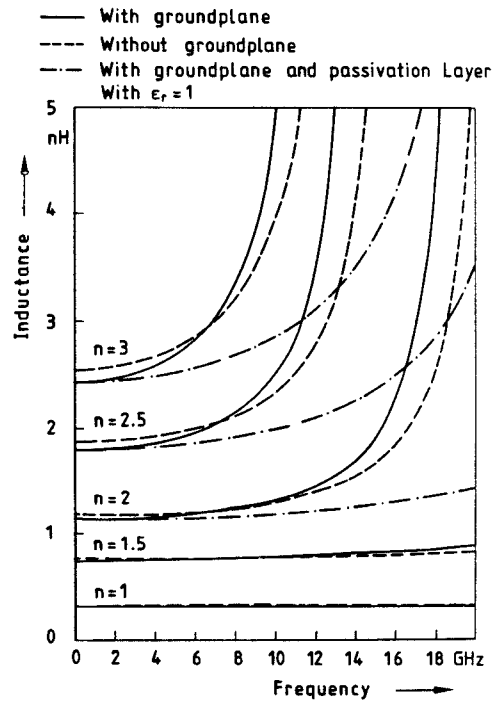


Fig.6: The frequency dependent inductance of a circular spiral inductor with and without ground metallization and with a passivation layer.

In Fig.6 the frequency dependent inductance with and without a ground metallization of the substrate is shown. Additionally the influence of a thin passivation layer (with an assumed dielectric constant of 1) on the frequency dependence is shown. It is assumed that the passivation layer is so thick, that the stray field between the circular line segments is only in this passivation layer. Under these assumption it can be recognized that a thin passivation layer drastically reduces the frequency dependence of the inductance.

The computation of the frequency dependent resistance of the spiral inductor is insofar a problem, as for metallization thicknesses between 2  $\mu\text{m}$  and 5  $\mu\text{m}$  the full skin effect theory has to be taken into account. An application of the incremental inductance rule /13/ normally leads to resistances which are too small. Therefore a closed formula has been developed which models the results of a rigorous theory /14/ for the skin effect resistance of a metal strip with rectangular cross section, and which also describes the resistance of the strip at low frequencies and for thin metallization thicknesses correctly.

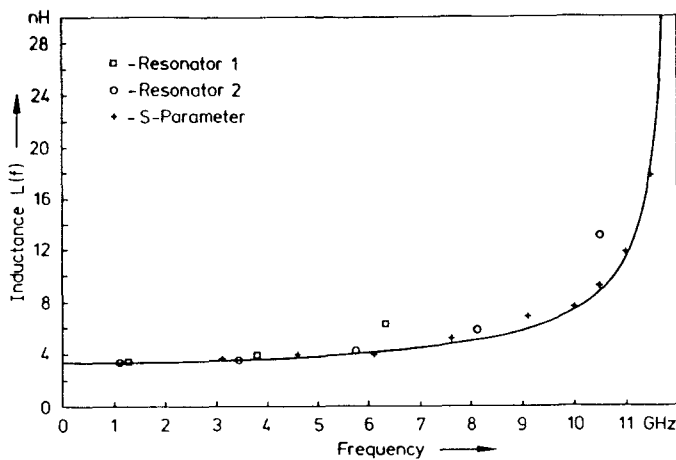


Fig.7: Comparison of the computed frequency depended inductance of a 3.5 turns circular spiral inductor with measured results.  
Parameters:  $r_1=57.5 \mu\text{m}$ ,  $w=20 \mu\text{m}$ ,  $s=15 \mu\text{m}$ ,  $h=150 \mu\text{m}$ ,  $t=5 \mu\text{m}$ .

First measurements made with the HP 8510 network analyzer and with a resonant measurement technique show (Fig.7) the good agreement as well of the static inductance as also of the resonant frequency of the model with the measurements. The measurements show that it is possible to model the complicate structure of the spiral inductor using accurate descriptions of the inductive and capacitive elements of the equivalent circuit. Especially the investigations show, that a computer model can be found, which is fast and therefore directly can be used in a workstation CAD program.

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